Exercise 307

A bacterial colony grown in a lab is known to double in number in 12 hours. Suppose, initially, there are 1000 bacteria present.

a. Use the exponential function $Q(t) = Q_0e^{kt}$ to determine the value k, which is the growth rate of the bacteria. Round to four decimal places.

b. Determine approximately how long it takes for 200,000 bacteria to grow.

Solution

Part (a)

Use the fact that the bacterial colony is known to double in number in 12 hours to determine k.

$$Q(t) = Q_0e^{kt}$$

$$2000 = 1000e^{k(12)}$$

Divide both sides by 1000.

$$2 = e^{12k}$$

Take the natural logarithm of both sides.

$$\ln 2 = \ln e^{12k}$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln 2 = 12k \ln e$$

Use the fact that $\ln e = 1$.

$$\ln 2 = 12k$$

Divide both sides by 12 to solve for k.

$$k = \frac{\ln 2}{12} \approx 0.0578$$

Part (b)

Plug in 200,000 for $Q(t)$, 1000 for Q_0, and the result for k from part (a).

$$Q(t) = Q_0e^{kt}$$

$$200{,}000 = 1000e^{0.0578t}$$

Divide both sides by 1000.

$$200 = e^{0.0578t}$$

Take the natural logarithm of both sides.

$$\ln 200 = \ln e^{0.0578t}$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln 200 = 0.0578t \ln e$$

Use the fact that $\ln e = 1$.

$$\ln 200 = 0.0578t$$

Divide both sides by 0.0578 to solve for t.

$$t = \frac{\ln 200}{0.0578} \approx 91.7$$

Therefore, it will take about 92 hours for the colony population to grow to 200,000 bacteria.